Interaction of Phonons and Spin Waves*

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Some difficulties in the interpretation of phonon-spin-wave interactions in magnetic films are considered. Experiments by various workers show that there is apparently a large difference in the wavelengths of the interacting phonon and spin waves. Furthermore, phonon excitation of the higher order spin waves is much stronger than expected; in some cases, the higher order spin waves are excited more strongly than are the lower order spin waves. It is shown that no difficulty exists when the spin-wave modes are Hermite functions, since then appreciable Fourier components of the spin-wave modes corresponding to the phonon wavelength exist. The strong dependence of the interaction on film thickenss is explained and the apparently erratic spin-wave-mode intensities appear reasonable on this basis.

INTRODUCTION

THE phonon-magnon interaction in thin magnetic films has been observed and studied by several groups.[1–4] Several aspects of these experiments remain without adequate explanation. The following two are considered here. For the case of the steady magnetic field, perpendicular to the film, the phonon wavelength appears to be much longer than the wavelength of the spin waves which it excites[2–4] and, in many cases, the higher order spin waves are more strongly excited by the phonons than the lower order spin waves.[3,4] We propose an explanation to these two apparent anomalies in terms of spin-wave modes which are Hermite functions rather than cosine functions.

Until recently it has been assumed that the standing spin-wave modes excited in a magnetic film by a uniform microwave field have been of the form of cosines,[5,6] taking the center of the film as the origin of the thickness coordinate. Recently, however, Portis[7] has suggested that the form of the standing spin wave may be quite different due to a variation, in the film, of the (z-directed, uniaxial) internal effective field.[7,8] This hypothesis is supported by considerable experimental evidence. It explains the tendency toward linear separation of spin-wave modes,[2,7,9] the anomalously large intensities of the higher order modes,[7,9] the "critical angle" effect,[8] and the general behavior of the spin-wave resonances as the steady magnetic field is turned out of the plane of the film.[8] This model will be applied here to the phonon-spin-wave interaction in magnetic films.

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CALCULATION OF POWER FLOW

We consider a magnetic film deposited on the end of a quartz rod. The film is coupled to the magnetic fields of a microwave cavity and provisions can also be made for direct piezoelectric excitation of phonons in the other end of the quartz rod. We wish to calculate the power flow for the processes:

Process I. Power from cavity to spin-wave modes (photon to magnon).
Process II. Power from spin-wave to phonon modes (magnon to phonon).
Process III. Power from phonon to spin-wave modes (phonon to magnon).
Process IV. Power from spin-wave modes to cavity (magnon to photon).

These are shown diagrammatically in Fig. 1. We assume that the interactions take place throughout the film and are not associated with the film surfaces.

The film is taken to lie in the xy plane with its surfaces at $z=\pm L/2$. The equations describing the interaction of the magnons with the phonons are given by Kittel[10] and we use them as written by Schlömann,[11] including the interaction with the cavity magnetic field.

$$\dot{m}=i\gamma[(H-H_{ex}a^2\partial^2/\partial z^2)m+b_2(\partial/\partial z)R-Mh] \quad (1a)$$

$$\rho\ddot{R}=\rho v^2(\partial^2/\partial z^2)R+(b_2/M)(\partial/\partial z)m. \quad (1b)$$

The magnetic field $H=H_{\text{applied}}-4\pi M$ is perpendicular to the plane of the film. H_{ex} is the exchange field, γ is the gyromagnetic ratio taken positive for electrons, a is the lattice constant, b_2 is a magnetoelastic coupling coefficient,[10] ρ is the density of the film, and v is the velocity of transverse phonons in the film. These quantities are assumed to be constant throughout the film. The displacement of an atom from its equilibrium position $R=R_x+iR_y$ is assumed to have the form $e^{i(\omega t-kz)}$ where k is the phonon wavelength; the microwave field $h=h_x+ih_y$ has the form $e^{i\omega t}$. The saturation magnetization M will be assumed to have the form (Portis[7])

$$M=M_0(1-4\epsilon z^2/L^2), \quad (2)$$

[10] C. Kittel, Phys. Rev. **110**, 836 (1958).
[11] E. Schlömann, J. Appl. Phys. **31**, 1647 (1960).

FIG. 1. Diagram of energy flow between the electromagnetic cavity mode, the spin-wave modes in the film, and the acoustic waves in the film.

where $\epsilon=\Delta M/M$ the fractional change of magnetization from 0 to $\pm L/2$. It is not necessary to have a variation of M throughout the film, but only a variation of the z directed, uniaxial, effective internal field. The form (2) is used for convenience. With this form for the magnetization, the homogeneous part of Eq. (1a) has the solution

$$m(z,t)=m_x+im_y=e^{i\omega t}\sum_n c_n\psi_n(z), \tag{3}$$

where the c_n are constants and

$$\psi_n(\xi)=e^{-\xi^2/2}H_n(\xi), \quad \xi=\alpha^{1/2}z \tag{4}$$

are the Hermite functions for a harmonic oscillator[7,12] with the usual transformation on z. The constant α is given by

$$\alpha=\frac{2}{aL}\left(\frac{4\pi M}{H_{ex}}\frac{\Delta M}{M_0}\right)^{1/2}, \tag{5}$$

where a small term in $\partial^2M/\partial z^2$ has been neglected.[7]

It is necessary for subsequent calculation to notice, as Kittel has pointed out,[10] that the term $(-b_2/M)\partial R/\partial z$ of Eq. (1a) represents an effective driving field on the magnetization due to the lattice displacement. Similarly the term $(ab_2/M)\partial m/\partial z$ represents a force per unit area on the lattice due to the magnons. This force/area tends to drive an xy plane of atoms transversely away from its equilibrium position.

The power per unit area of film will now be calculated for each of the processes I, II, III, and IV, using Eqs. (1) with the wave functions given above.

Process I: Photon to Magnon

Consider Eq. (1a) in which R is zero and the term in h is a small perturbation. The spin waves will then have the form (3) and Eq. (1a) becomes

$$\sum_n(\omega+i\tau_n^{-1}-\omega')c_n\psi_n(z)=-\gamma M(z)h, \tag{6}$$

where the relaxation time τ_n of the nth mode has been introduced and $\omega'=\gamma(H-H_{ex}a^2\partial^2/\partial z^2)$. If the field, $H_{applied}$, is adjusted to resonance $(\omega-\omega'_n)\psi_n(z)=0$, since $\psi_n(z)$ is a solution, so that $i\tau_n^{-1}$ only remains in the parenthesis on the left side. Both sides of Eq. (6) are then multiplied by $c_m^*\psi_m(z)$ and integrated from $-L/2$ to $+L/2$. From the value of α which will be introduced later one can find that the functions $\psi_n(\xi)$ become negligible at $\pm L/2$ unless the mode number n is large. Therefore, $\pm L/2$ will be replaced by $\pm\infty$. The amplitude of the nth spin-wave mode is then

$$c_n=iN_n^2\tau_n\gamma M_0h\int_{-\infty}^{+\infty}\psi_n(z)dz, \tag{7}$$

where the orthonormality of the Hermite functions has been used and

$$N_n^2=(\alpha/\pi)^{1/2}(1/2^nn!) \tag{8}$$

is the usual normalization factor.[12] We will restrict ourselves to small variations in M so that on the right side of Eq. (7), $M\approx M_0$. From (7) it is seen that only even modes can be excited by a uniform h field since the Hermite functions $\psi_n(z)$ are even for even n and odd for odd n.

In order to present the results in consistent form for all four processes, I, II, III, and IV, the Fourier transform of the Hermite functions[13]

$$g_n(K)=\int_{-\infty}^{+\infty}\psi_n(\xi)e^{iK\xi}d\xi=i^n(2\pi)^{1/2}\psi_n(K) \tag{9}$$

is introduced, where $\xi=\alpha^{1/2}z$ and $K=k/\alpha^{1/2}$. In terms of (9) the amplitude coefficients (7) become

$$c_n=i\tau_n\gamma M_0hg_n(0)/2^nn!\pi^{1/2}, \quad n=(0,2,4,\cdots) \tag{10}$$

where (8) has been used to eliminate N_n.

The power absorbed by the nth mode, per unit area of the film, is

$$P_I=\left|\frac{1}{8\pi}\frac{\omega}{2\pi}\int_{-\infty}^{+\infty}\int_0^T h(t)\frac{dm^*(z,t)}{dt}dtdz\right|, \tag{11}$$

where $\pm L/2$ have been replaced by $\pm\infty$. By use of (3), (9), and (10), and remembering that $h\propto e^{i\omega t}$, the power absorbed is found to be

$$P_I=\frac{\omega\tau_n\gamma M_0h^2[\psi_n(0)]^2}{4(\pi\alpha)^{1/2}2^nn!} \quad (n=0,2,4,\cdots). \tag{12}$$

Except for the mode $n=0$, which is eddy-current broadened, the relaxation time is constant to within 10% from line to line so that the only part of this expression which is strongly dependent on n is

$$P_I\propto[\psi_n(0)]^2/2^nn! \quad (n=0,2,4,\cdots). \tag{13}$$

This is identical to the result of Portis.[7]

Process II: Magnon to Phonon

For this case we assume phonons of the form

$$R=\sum_k R_ke^{i(\omega t-kz)} \tag{14}$$

[12] L. Pauling and E. B. Wilson, Jr., *Introduction to Quantum Mechanics* (McGraw-Hill Book Company, Inc., New York, 1935), Chap. 3; Leonard I. Schiff, *Quantum Mechanics* (McGraw-Hill Book Company, Inc., New York, 1949), Chap. 4.

[13] E. C. Titchmarsh, *Introduction to the Theory of Fourier Integrals* (Oxford University Press, Oxford, 1948), 2nd ed., p. 81.

which are being perturbed slightly by the spin-wave term $(b_2/M)\partial m/\partial z$ of Eq. (1b). Proceeding as before, we have from Eq. (1b)

$$\rho\sum_k(v^2k^2-2i\omega\tau_k^{-1}-\omega^2)R_ke^{-ikz} = (b_2/M)c_n\partial\psi_n(z)/\partial z, \quad (15)$$

where the relaxation time for phonons τ_k has been introduced by letting $\omega\rightarrow\omega+i\tau_k^{-1}$ and neglecting τ_k^{-2} as a small quantity.[14] We again assume $M\cong M_0$ on the right side. Multiplying both sides by e^{ikz} and integrating from $-L/2$ to $+L/2$ we find

$$R_k=\frac{i\tau_kb_2c_n}{2\omega L\rho M_0}\int_{-L/2}^{+L/2}\frac{d\psi_n(z)}{dz}e^{ikz}dz. \quad (16)$$

Integrating by parts and remembering that $\psi_n(\pm L/2)\approx 0$, the amplitude of the kth phonon is found to be

$$R_k=\frac{b_2\tau_kkg_n(K)c_n}{2\omega L\rho M_0\alpha^{1/2}} \quad (n=0, 2, 4, \cdots). \quad (17)$$

The power from the spin waves to the phonons is

$$P_{\mathrm{II}}=\left|\frac{\omega}{2\pi L}\int_{-L/2}^{+L/2}\int_0^T\left(\frac{ab_2}{M_0}\frac{\partial m(z,t)}{\partial z}\right)\frac{\partial R^*(z,t)}{\partial t}dt\,dz\right|, \quad (18)$$

where the term $(ab_2/M_0)(\partial m/\partial z)$ is the appropriate force/area as discussed above. By inserting (3) and (14) into (18), and using (9), the power flow from the nth spin-wave mode to the kth acoustic mode is found.

$$P_{\mathrm{II}}=\frac{\pi ab_2^2k^2\tau_k\eta^2[\psi_n(K)]^2}{(\pi\alpha)^{1/2}\rho L2^nn!} \quad (n=0, 2, 4, \cdots). \quad (19)$$

The amplitudes c_n have been set by the normalization condition

$$\frac{1}{L}\int_{-\infty}^{+\infty}c_nc_n^*[\psi_n(z)]^2dz=\frac{c_nc_n^*}{LN_n^2}=\eta^2M_0^2, \quad (20)$$

where η is a small number. Here again the only term strongly dependent on n is $[\psi_n(K)]^2/2^nn!$.

Process III: Phonon to Magnon

The spin-wave excitations are determined from (1a) with $h=0$ and the term in R assumed small so that the modes are essentially pure spin-wave modes which are being excited by a phonon flux. Equation (1a) becomes

$$\sum_n(\omega+i\tau_n^{-1}-\omega')c_n\psi_n(z)=-ik\gamma b_2R_ke^{-ikz}, \quad (21)$$

for the kth phonon. Proceeding in the standard way we find

$$c_n=\frac{\tau_n\gamma b_2kR_kg_n(K)}{\pi^{1/2}2^nn!} \quad (n=0, 2, 4, \cdots). \quad (22)$$

(We arbitrarily restrict n to even values here since, only even spin-wave modes excite the cavity.) Using the effective field due to phonons $(-b_2/M)\partial R/\partial z)$ in Eq. (11) we find

$$P_{\mathrm{III}}=\frac{\pi\omega\tau_nk^2R_k^2\gamma^2b_2^2[\psi_n(K)]^2}{(\pi\alpha)^{1/2}(4\pi\gamma M_0)2^nn!} \quad (n=0, 2, 4, \cdots). \quad (23)$$

Since τ_n is a weak function of n, P_{III} depends on n mainly through $[\psi_n(K)]^2/2^nn!$. Therefore, P_{III} has the same n dependence as P_{II}.

Process IV: Magnon to Photon

The microwave magnetic field generated by the spin-wave mode n is

$$h\propto\frac{1}{L}\int_{-L/2}^{+L/2}c_n\psi_n(z)dz\propto c_ng_n(0).$$

The power into the cavity is

$$P_{\mathrm{IV}}\propto h^2\propto[c_n\psi_n(0)]^2\propto[\psi_n(0)]^2/2^nn! \quad (n=0, 2, 4, \cdots), \quad (24)$$

when the normalization (20) is carried out. Therefore, P_{IV} has the same n dependence as P_{I}.

Phonon Boundary Conditions

In the preceding analysis the effects of the boundaries $z=\pm L/2$ on the form of the phonon wave functions have been neglected. We now take into account the metal-air boundary at $z=-L/2$ and the metal-quartz boundary at $z=+L/2$. The appropriate sound velocity in quartz is 5.4×10^5 cm/sec and in nickel (which we take to be reasonably similar to permalloy) it is 3.0×10^5 cm/sec. This gives a theoretical reflection coefficient of 0.08. We will, therefore, assume the medium to be continuous acoustically at the permalloy-quartz interface. At the air-metal surface $(z=-L/2)$ the reflection coefficient will be unity and an antinode of the phonon wave function will exist there. The phonon wave function will be

$$R=e^{i\omega t}\sum_k R_k\cos k(z+L/2), \quad (25)$$

which satisfies the foregoing requirements.

The power flow for processes II and III can now be recalculated using this form for the phonons. Following the same procedures as before but using Eq. (25) for the phonons, we obtain

$$R_k=-i\frac{b_2\tau_kkg_n(K)c_n}{\omega L\rho M_0\alpha^{1/2}}\left[\frac{kL\sin(kL/2)}{kL+\sin kL\cos kL}\right] \quad (n=0, 2, 4, \cdots) \quad (26)$$

[14] The elastic constant c_{44} and density, and therefore v, are assumed independent of z. This is a good approximation for small variations in the composition of permalloy since these quantities are quite similar for both iron and nickel.

for the amplitude of the phonons excited by the spin waves. The power is

$$P_{\mathrm{II}}=\frac{\pi a b_2^2 k^2 \tau_k \eta^2[\psi_n(K)]^2}{(\pi\alpha)^{1/2}\rho L 2^n n!}\times\left[\frac{2kL\sin^2(kL/2)}{kL+\sin kL\cos kL}\right]$$
$$(n=0, 2, 4, \cdots), \quad (27)$$

which is identical to Eq. (19) except for the factor in square brackets. For a given k the dependence on n is unchanged.

Similarly, for process III we obtained for the amplitude of spin waves excited by phonons

$$c_n=-i\frac{\tau_n\gamma b_2 k R_k g_n(K)}{\pi^{1/2}2^n n!}[\sin(kL/2)]$$
$$(n=0, 2, 4, \cdots). \quad (28)$$

The power for process III is

$$P_{\mathrm{III}}=\frac{\pi\omega\tau_n k^2 R_k^2\gamma^2 b_2^2[\psi_n(K)]^2}{(\pi\alpha)^{1/2}(4\pi\gamma M_0)2^n n!}[2\sin^2(kL/2)]$$
$$(n=0, 2, 4, \cdots). \quad (29)$$

This is identical to the previous expression, Eq. (23), except for the factor in brackets. Again, the dependence on n is unchanged. The extra term in Eqs. (27) and (29) shows that, for $kL=p\pi$ where p is an odd integer, the power is doubled due to constructive interference from the free surface of the film. When p is an even integer the power is zero due to destructive interference from the free surface. If other boundary conditions than these are imposed on the phonons, one would obtain different interference terms. The dependence of the spectrum on the spin-wave-function index n, however, would not change. This can be seen by carrying out the calculation with an arbitrary combination of sine and cosine phonon wave functions with arbitrary phases. It is not hard to see that the form of Eqs. (26) or (29), in which the interference term and the term containing n occur as separate factors, will always be obtained provided the phonon wave function can be separated into a time and a space factor.

Combinations of I, II, III, and IV

In the experiments of Pomerantz[2] the cavity is excited by spin waves which, in turn, are excited by independently generated phonons. The intensities of excitation of the cavity are then given by the product of process III and IV (n dependence only).

$$I_n^{(1)}(K)\propto[\psi_n(0)\psi_n(K)/2^n n!]^2 \quad (n=0, 2, 4, \cdots). \quad (30)$$

In the experiments of Lewis, Philips, and Rosenberg[4] and of Seavey[3] the spin waves are excited by the cavity which in turn excites phonons. The phonons travel to the opposite end of the quartz rod, are reflected, and re-excite the spin waves which finally excite the cavity. The intensities observed are found from the product of processes I, II, III, and IV

$$I_n^{(2)}(K)\propto[\psi_n(0)\psi_n(K)/2^n n!]^4, \quad (n=0, 2, 4, \cdots). \quad (31)$$

Formulas (30) and (31) are valid for all n except $n=0$, for which eddy-current damping is appreciable,[9] and large n, for which Hermite functions are no longer the proper wave functions.[7]

The first few functions $[\psi_n(K)]^2/2^n n!$ are listed in Appendix A.

COMPARISON WITH EXPERIMENTAL RESULTS

Let us first consider the experiment of Pomerantz[2] on permalloy films. In order to fit his mode positions to a square law $H_n\propto n^2$, he labels them $n=0$, 9, 11, 13, 15, and 17. This indicates that they actually go quite linearly, that is, they could be labeled $n=0$, 2, 4, $\cdots$ with $H_n\propto n$ without serious error. The spin-wave functions are, therefore, likely to be reasonably close to Hermite functions and the foregoing theory is applicable.

In order to calculate the intensities $I_n^{(1)}(K)$ the value of $K=k/\alpha^{1/2}$ must be determined. Pomerantz gives the values $L=2600$ Å, and $k\approx 2\times 10^5$ cm^{-1}. We take $4\pi M=10^4$ G, $a=2.5\times 10^{-8}$ cm and $H_{\mathrm{ex}}=2A/Ma^2=4\times 10^6$ Oe. The value of the exchange constant A is taken as 10^{-6} erg/cm. Portis finds the value $\Delta M/M\approx 0.08$ for a film[7] and we will use this value. The reduced phonon propagation vector then becomes $K\approx 1$. The comparison of calculated and experimental values of $I_n^{(1)}(K)$ is made in Table I. The higher orders are excited appreciably because the spin-wave functions have appreciable Fourier components at the phonon wave length.

We next consider the experiments of Seavey[3] on permalloy. His measured-mode positions are nearly linear and the above theory should apply. In order to compare Seavey's results (his Fig. 6) to the expression

TABLE I. Comparison of theory and experiment for the intensities observed by Pomerantz (Ref. 2) and by Seavey (Ref. 3). Pomerantz performed a two-step (III and IV of Fig. 1) experiment on a 2600 Å, permalloy (80–20) film at 8.9 kMc/sec with $K\approx 1$. Seavey performed a four-step (I, II, III, and IV of Fig. 1) experiment on $a\approx 4500$ Å permalloy (88–12) film at X band with $K\approx 1.3$. Normalized to unity for $n=0$.

	Theory	Experiment Pomerantz	Theory	Experiment Seavey
n	$I_n^{(1)}(1)$	$I_n^{(1)}(1)$	$I_n^{(2)}(1.3)$	$I_n^{(2)}(1.3)$
0	1.00	1.00	1.00	1.00
2	0.25	0.25	2.35	1.08
4	0.39	0.12	0.27	1.98
6	0.23	0.05	~0	1.78
8	0.07		3.90	1.16
10			0.08	0.22

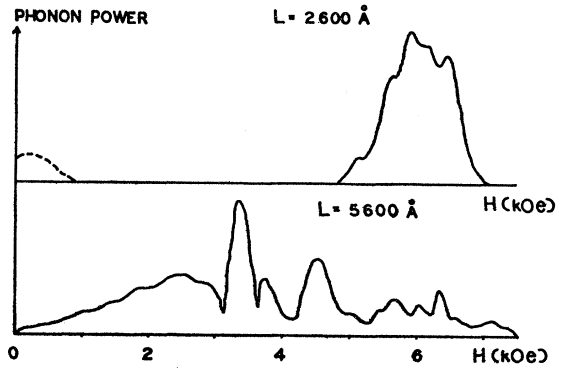

FIG. 2. Plot of cavity power against steady magnetic field for two nickel films when $\mathbf{k}\|\mathbf{M}\|z$, from Lewis, Philips, and Rosenberg (Ref. 4). Their mode labeling has been omitted here since it is apparently based on the hypothesis of a uniform film with perfect surface-spin pinning. Although the ordinate is labeled "phonon power" the description of the experiment makes it evident that it is actually "cavity power."

for the intensities $I_n^{(2)}(K)$, we use the same parameter values as used for Pomerantz's experiment, except for the film thickness which is $L\approx 4500$ Å. This gives $K\approx 1.3$. The comparison of theory and experiment is given in Table I.

The agreement of experiment with theory is generally within an order of magnitude. It appears to be possible to obtain somewhat better agreement just by careful adjustment of the value of K. It is likely that the remaining differences between theory and experiment are due to the spin-wave modes having mixtures of other functions, i.e., sinusoidal, with the Hermite functions. As the films are made more uniform with respect to the thickness dimension the Hermite spin-wave functions should be replaced by cosine functions provided some means of spin pinning still remains at the surfaces. Under the assumption of pure cosine spin-wave modes it would not be possible to get a phonon-spin-wave interaction at all, except for the lowest modes of thick films or for multiple phonon-magnon processes. It, therefore, appears that nonuniform films are responsible for the large spin-wave-phonon interactions which are observed for the higher spin-wave modes.

The apparently erratic intensity variations which occur are a consequence of the rapid variation of the Hermite functions with respect to either the reduced wave vector K (given n) or the index n (given K). These variations are amplified enormously in the experiments in which several of the processes I, II, III, or IV occur consecutively. This behavior is shown by the results of Lewis, Philips, and Rosenberg.[4] Their results of measurements on nickel, made in the same way as those of Seavey on permalloy, are shown in Fig. 2. Several qualitative comparisons to theory can be made. (1) The intensities are *apparently* erratic, in agreement with the foregoing discussion. (2) Some modes are missing or very small due to the zeros in the Hermite functions. (3) For the 2600 Å film, Fourier components

TABLE II. Theoretical dependence of mode intensities on the film thickness for the two-step experiment, processes III and IV. The $I_n^{(1)}(K)$ are normalized to 1.00 for $n=0$. For the four-step process the values would be squared.

n	$L=2600$ Å $I_n^{(1)}(1)$	$L=5200$ Å $I_n^{(1)}(\sqrt{2})$	$L=10\,400$ Å $I_n^{(1)}(2)$
0	1.00	1.00	1.00
2	0.25	2.25	12.3
4	0.39	0.39	5.6
6	0.23	0.05	4.6
8	0.07	0.43	0.02

of the Hermite spin-wave functions, corresponding to the phonon wavelength, are weak for the higher spin-wave modes and, therefore, no appreciable excitation is observed. For the 5600 Å film, however, the appropriate Fourier components are large and appreciable excitation of the higher-order spin waves is observed. The expressions for the mode intensities, Eqs. (30) and (31), give this behavior quantitatively through the dependence of K on the film thickness ($K=k/\alpha^{1/2}\propto L^{1/2}$). A set of values calculated from Eq. (30) is given in Table II to illustrate this thickness dependence.

The "highly individual behavior" of nickel films and the frequent "second burst of phonon power" occurring at higher mode numbers mentioned by Lewis, Philips, and Rosenberg[4] are understandable in terms of the rapid variations of the Hermite functions as the film parameters are changed. In addition to being sensitive to the film thickness, the spectrum will be sensitive to changes in the magnetization variation[15] throughout the thickness of the film. Even though this enters K as $(M_0/\Delta M)^{1/4}$ its effect may be considerable due to the strong dependence of the intensities on K. As Portis has pointed out,[7] if ΔM becomes sufficiently small the Hermite spin-wave functions will no longer be the correct functions. One should take care not to apply the results given here unless $H_n\propto n$, a situation which is reasonably correct for many films. Furthermore, it should be remembered that the results given here are for weak coupling only. The phonon-magnon coupling of permalloy near its zero magnetostriction composition is presumed to be weak.

The spectrum should also be strongly dependent on the phonon wavelength. Since this is fixed by the microwave frequency one would expect the spectrum to be strongly frequency dependent.

CONCLUSION

The apparent discrepancy between phonon and magnon wavelengths in magnon-phonon interactions in magnetic films is removed if the spin waves are taken to be Hermite functions. Appreciable Fourier compo-

[15] It should be emphasized that this magnetization variation is simply a convenient way of introducing a z dependence of the z directed, uniaxial, internal field. Other mechanisms such as a variation of strain throughout the thickness of the film may actually be operative.

nents of the spin wave then exist corresponding to the phonon wavelength and the calculated excitation of the spin waves by the phonons comes out in good agreement with experiment. The dependence of the spectrum on film thickness and the apparently erratic intensities are understandable on this basis.

The effects of eddy currents are quantitatively neglected but must be taken into account for the $n=0$ mode and perhaps for higher-order modes if the film thickness is increased past 5000 Å. It is apparent that the spin-wave functions will not be exactly Hermite functions in all cases because of variations in film fabrication. It appears that more exact comparison between experiment and theory will be possible if efforts are made to produce either perfectly uniform films or films in which the internal effective field varies exactly parabolically.

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APPENDIX I: THE FUNCTIONS $J_n(K)=[\psi_n(K)]^2/2^n n!$ WHERE $\psi_n(K)=\exp(-K^2)H_n(K)$.

$$J_0(K)=\exp(-K^2),$$

$$J_2(K)=\exp(-K^2)(2K^2-1)^2/2,$$

$$J_4(K)=\exp(-K^2)(4K^4-12K^2+3)^2/24,$$

$$J_6(K)=\exp(-K^2)(8K^6-60K^4+90K^2-15)^2/720,$$

$$J_8(K)=\exp(-K^2)(16K^8-224K^6+840K^4-840K^2+105)^2/40\,320.$$

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Excited States of the *F* Center

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Various *L* bands have been measured in KCl and KBr x rayed at liquid-helium temperature and in KCl:H x rayed at room temperature. These results argue against the suggestion that the *L* bands arise from complexes involving *F* centers and support the argument that the *L* bands arise from the *F* center itself. A charge-transfer model of the *L* bands is proposed in which the transitions correspond to a combination of transferring the *F*-center electron to a neighboring alkali ion and raising the resulting alkali atom to one of its excited states.

INTRODUCTION

FOR many years the picture of the *F* center in alkali halides has seemed reasonably complete.[1] The center consists of an electron at a negative ion vacancy. The principal electronic transition is seen in absorption as the *F* band which raises the electron to an energy level a few tenths of an electron volt below the conduction band. A weaker transition on the high-energy side of the *F* band is known as the *K* band. It also arises from the *F* center and is thought to involve excitation of the electron either into the conduction band or very close to it.

A startling development, due to Lüty, has been injected into this picture.[2] In additively colored potassium and rubidium salts he has found three new bands on the high-energy side of the *F* and *K* bands. These bands, called L_1, L_2, and L_3 bands, are smaller than the *F* band by one to two orders of magnitude but are proportional to the *F* band. The peak positions of the *L* bands vary as the host lattice is changed and follow the empirical Mollwo-Ivey relation[3] as do the *F* and *K* bands. This relationship is

$$\nu d^n=\text{const}, \tag{1}$$

where ν is the frequency of the maximum of the band, d is the lattice constant of the host material, and n is a constant which is nearly 2.

The most surprising feature of these bands is that they lie from about 0.7 to 2.5 eV higher in energy than the *F* band. From all that is known about the *F* center, these transitions would be to states well within the conduction band if the *F* center is the defect responsible for the bands. One might expect that photoconductivity would be observed on irradiation into these bands; this had indeed been observed by Inchauspé before the discovery of the *L* bands.[4] In KBr at 80°K, Inchauspé found photoconductive peaks at the L_2 and L_3 positions and has found another peak at even higher energies which has not been identified in optical absorption. Wild and Brown have examined the photoconductivity

[1] For a recent review of the properties of the *F* center see J. H. Schulman and W. D. Compton, *Color Centers in Solids* (Pergamon Press Inc., New York, 1962).
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[4] N. Inchauspé, Phys. Rev. **106**, 898 (1957).